Capacitance–Voltage Characteristics of Microwave Schottky Diodes

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***Abstract*—The capacitance of small-area microwave Schottky diodes is strongly affected by the edge effect, which is not adequately described by existing analytical models. Based on an analytical solution of Poisson's equation, we calculated capacitances of metal circular dots and metal stripes on the surface of a doped semiconductor material. When the dimensions of the dot or stripe are much larger than the depletion region, the results are reduced to the conventional formula for a parallel-plate capacitor. In the opposite limit, the overall capacitance is determined by the edge effect. This edge capacitance is proportional to the device periphery, with the coefficient of proportionality dependent on the shape of the metal. In the most interesting case of a round metal dot, the edge capacitance is given by $C = 4\epsilon a$, where ϵ is the dielectric permittivity of the semiconductor and a is the radius of the metal dot. The parallel-plate component of the device capacitance is modulated by the applied voltage; the edge component is nearly independent of the applied voltage. Hence, the largest capacitance modulation is achieved in devices with the smallest ratio of the device periphery over the device area, which has the smallest edge effect. The measured capacitances of small round GaAs Schottky barrier diodes are in reasonable agreement with the results of our calculation.**

I. Introduction

As device sizes are scaled down, the edge effects play an increasingly important role in determining the capacitance of semiconductor devices with Schottky barriers [1]. At the same time, existing analytical models are based on the equation for a parallel-plate capacitance (with some corrections) and they only adequately describe large-area devices.

Wasserstrom and McKenna [2] and Copeland [3] reported the results of numerical calculations of the capacitances of metal dots and stripes on a semiconductor surface. They also proposed an analytical interpolation formula which is valid for relatively large area devices when the characteristic size, a, of the metal contact is much larger than the depth of the depletion region, R_1.

In this paper, we find analytical solutions for a metal hemisphere, a metal semicylinder, a metal stripe, and a metal ellipsoid for a limiting case of a very small ellipsoid (with dimensions smaller than the depth of the depletion layer, R_1). In the limiting case $a_1, a_2 \gg a_3$, where a_1 and a_2 are ellipsoid axes in the plane of the semiconductor surface and a_3 is an ellipsoid axis perpendicular to the semiconductor surface, this geometry corresponds to an important limiting case of an elliptical metal dot on the semiconductor surface (or a round metal dot when $a_1 = a_2$). In the opposite limiting case ($a_1, a_2 \ll a_3$), this solution describes a conductive needle piercing the semiconductor material. Our analytical solutions allows us to propose interpolation formulas for practical geometries of a metal dot and a metal stripe which are accurate in two limiting cases of large and small characteristic metal sizes ($a \gg R_1$ and $a \ll R_1$) and give reasonable results for intermediate case when a is of the order of R_1.

II. Basic Assumptions

We assume that the semiconductor is doped with shallow ionized donors with concentration N_d. The dielectric permittivity, ϵ, of the semiconductor is much larger than the dielectric permittivity of vacuum. This allows us to neglect the normal component of the electric field at the semiconductor interface.

Our calculations involve the solution of Poisson's equation for different geometries in spherical and cylindrical coordinate systems:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rU) = -\frac{qN_d}{\epsilon} \tag{1}$$

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = -\frac{qN_d}{\epsilon}. \tag{2}$$

Here U is the electric potential and q is the electronic charge. Once Poisson's equations are solved, we can find the total charge in the depletion layer, Q. The voltage drop across the depletion region is given by

$$U_1 = U(R_1) - U(a) \tag{3}$$

where R_1 is the radius of the depletion region, r is the distance from the center of the contact, a is the radius of the hemisphere, and $U(a)$ is the semiconductor potential at the boundary with the metal. (We choose $U(a)$ as a

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reference point, i.e., $U(a)=0$).) The applied bias is given by

$$V_{\text{applied}} = V_{\text{bi}} - U_1 \tag{4}$$

where V_{bi} is the built-in voltage of the Schottky barrier (a positive sign of V_{applied} corresponds to the forward bias).

III. Capacitance of a Hemispherical Schottky Diode

For a spherical Schottky diode, the depletion layer is a hemisphere. Integrating (1) and taking into account that $\partial U/\partial r|_{r=R_1}=0$, we find the potential distribution in the depletion layer:

$$U = qN_d\left(a^2 - r^2 - 2R_1^3/r + 2R_1^3/a\right)/(6\epsilon). \tag{5}$$

From (5), we find the voltage drop across the depletion layer:

$$U_1 = qN_d a^2(x-1)^2(2x+1)/(6\epsilon) \tag{6}$$

where $x = R_1/a$. The total charge in the depletion layer is equal to

$$Q = 2\pi q N_d\left(R_1^3 - a^3\right)/3. \tag{7}$$

Hence, the device capacitance, C_s, can be calculated as follows:

$$C_s = (dQ/dR_1)/(dU_1/dR_1) = 2\pi\epsilon a R_1/(R_1 - a). \tag{8}$$

This expression is identical to the expression for a capacitance of two hemispherical electrodes of radii a and R_1 separated by a dielectric. The presence of the depleted donors does not change the formula for the capacitance. (The donor concentration, N_d, determines R_1.) A similar situation exists for a planar Schottky diode.

Equation (8) allows us to determine the capacitance variation with size. When $R_1 \gg a$,

$$C_s \approx 2\pi\epsilon a = C_{\text{edge}} \tag{9}$$

i.e., C_s is proportional to the contact perimeter. In the opposite limiting case, when $R_1 - a \ll a$, the conventional parallel-plate capacitance formula is applicable, as expected:

$$C_s \approx 2\pi\epsilon a^2/(R_1 - a) = \epsilon S/d = C_{\text{planar}}. \tag{10}$$

Here S is the contact area and d is the width of the depletion layer. Fig. 1 shows the variation of C_s/C_{edge} versus R_1/a. As can be seen from the figure, $C_s \to C_{\text{edge}}$ when $R_1/a > 4$.

For any a/R_1 ratio, C_s can be presented as the sum of the planar and edge capacitances:

$$C_s = C_{\text{planar}} + C_{\text{edge}}. \tag{11}$$

As will be shown below, for $R_1 - a \ll a$, this equation applies for other geometries, such as a semicylinder (see Section IV), a round dot, or a metal stripe, if we denote

$$C_{\text{edge}} = \beta\epsilon P \tag{12}$$

where P is the perimeter of the metal edge and β is a numerical coefficient of the order of unity which primarily depends on the surface curvature.

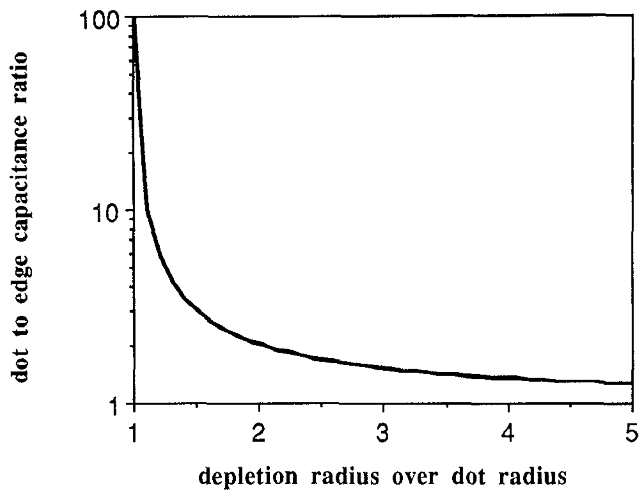


Fig. 1. Variation of C_s/C_{edge} versus R_1/a.

IV. Capacitance of a Semicylinder Schottky Diode

For this geometry (see Table I), the depletion layer is a semicylinder. Integrating (2), we obtain

$$U = qN_d\left(a^2 - r^2 + 2R_1^2\ln(r/a)\right)/(4\epsilon). \tag{13}$$

From (13), we find the voltage drop across the depletion layer:

$$U_1 = qN_d\left(a^2 - R_1^2 + 2R_1^2\ln x\right)/(4\epsilon). \tag{14}$$

The total charge in the depletion layer is equal to

$$Q = qN_d\pi W\left(R_1^2 - a^2\right)/2 \tag{15}$$

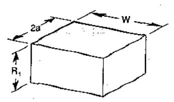
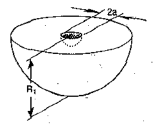
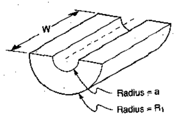
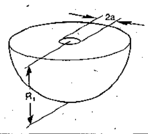
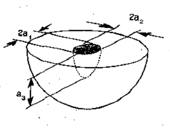
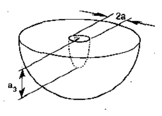
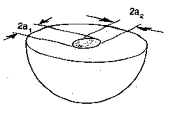
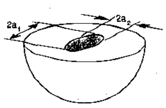
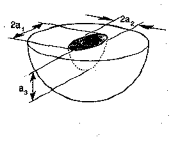
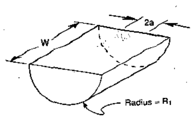
where W is the length of the cylinder, which is assumed to be much larger than R_1. Hence, the device capacitance

$$C = (dQ/dR_1)/(dU_1/dR_1) = \epsilon\pi W/(\ln x). \tag{16}$$

This expression is identical to the expression for a capacitance of two semicylindrical electrodes of radii a and R_1 separated by a dielectric; i.e., the presence of the depleted donors does not change the formula for the capacitance. (The donor concentration, N_d, determines R_1.) This is similar to the case of a hemisphere or to the case of a planar capacitance.

In the limiting case when $R_1 - a \ll a$, the logarithm expansion into the Taylor series in (16) yields (11) with $\beta = \pi/4$. As was shown in Section II, $\beta = 1$ for a hemisphere. The results of the numerical calculations by Copeland [2] for large-area metal dots and by Wasserstrom and McKenna [3] for large-area metal stripes are described by (11) with $\beta = 0.75$ and $\beta = 0.708$, respectively. Hence, we expect that (12) with $\beta \approx 0.71 - 0.75$ is applicable to any flat geometry as long as $R_1 - a \ll a$. For a curved surface, β changes to $\pi/4$ for a semicylinder and to 1 for a hemisphere; i.e., β increases with the surface curvature.

TABLE I
CAPACITANCES OF SCHOTTKY DIODES OF DIFFERENT GEOMETRIES

Geometry	Notation	Capacitance
planar		$C = 2\varepsilon a W/R_1$
hemisphere		$C = 2\pi\varepsilon a R_1/(R_1 - a)$
semi-cylinder		$C = \pi\varepsilon W/\ln(R_1/a)$
round dot		$C = 4\varepsilon a$
semi-ellipsoid		$C = \dfrac{2\pi\varepsilon(a_1^2 - a_2^2)^{1/2}}{F\{\mathrm{Cos}^{-1}(a_3/a_1), [(a_1^2-a_2^2)/(a_1^2 - a_3^2)]^{1/2}\}}$
semi-ellipsoid with round cross-section		$C = \dfrac{2\pi\varepsilon(a^2 - a_3^2)^{1/2}}{\mathrm{Cos}^{-1}(a_3/a)}$
elliptic metal dot		$C = \dfrac{2\pi\varepsilon a_1}{K([1 - (a_2/a_1)^2]^{1/2})}$
elongated elliptic dot		$C = \dfrac{2\pi\varepsilon a_1}{\ln(4a_1/a_2)}$
elongated semi-ellipsoid		$C=2\pi\varepsilon a_3/\ln[4a_3/(a_1+a_2)]$
narrow metal stripe		$C = \pi\varepsilon W/\ln(2R_1\sqrt{e}/a)$

The edge capacitance, $C_{\mathrm{edge}} = \beta\epsilon P$, where P is a perimeter and β varies between 0.7 and 1 depending on geometry.

A conventional approach is to interpolate the shape of the depletion region near the edges of the metal stripe by quarter cylinders. This is equivalent to introducing an additional edge charge:

$$Q_{\mathrm{edge}} = 2qN_dW(\pi R_1^2/4). \tag{17}$$

This charge leads to the capacitance:

$$C_{\mathrm{edge}} = (dQ_{\mathrm{edge}}/dR_1)/(dU_1/dR_1) = \pi\epsilon W \tag{18}$$

which is approximately twice as large as the result predicted by (12). (This disagreement was already noticed to Copeland [2].)

V. CAPACITANCE OF A METAL SEMIELLIPSOID EMBEDDED INTO SEMICONDUCTOR

Different geometries of Schottky diodes are limiting cases of a semiellipsoid with axes a_1, a_2, and a_3. For example, an elliptical dot on a semiconductor surface corresponds to $a_3 = 0$ and a round dot corresponds to $a_3 = 0$, $a_1 = a_2$. We are able to find an analytical solution for an ellipsoid only in a limiting case of a large depletion layer ($R_1 \gg a_1, a_2, a_3$). In this case, the boundary of the depletion layer is still very close to a hemisphere, and the total charge in the depletion layer is approximately given by

$$Q = 2\pi qN_dR_1^3/3. \tag{19}$$

Hence, far from the metal ellipsoid ($r \gg a_1, a_2, a_3$), the electric potential is approximately the same as for a hemisphere:

$$U = (-2Q/r - 2\pi qN_dr^2/3)/(4\pi\epsilon) + U_2. \tag{20}$$

The first and second terms in the parenthesis of (20) describe the contribution from the charge, Q, on the metal surface and the contribution from the charge in the depletion layer within a hemisphere of radius r, respectively, and U_2 is a constant to be found. At distances r close to the metal, the depletion charge affects the surface distribution of the electric field only slightly. For this to be true, the charge contained in the depletion layer within a hemisphere of radius r must be much smaller than the total charge, Q, i.e., for $r \ll R_1$. Therefore, very close to the surface, the solution coincides with that for an ellipsoid in a dielectric medium without any depletion charges. In the intermediate region, $a_1, a_2, a_3 \ll r \ll R_1$, the second term in the parentheses on the right-hand side of (20) is still much smaller than the first one. This means that we have to find a solution which coincides with that for an ellipsoid in a dielectric medium for $r \ll R_1$ and asymptotically approaches the $1/r$ dependence for large values of r. The charge distribution, σ, on the surface of a metal ellipsoid in a dielectric medium is given by [4]

$$\sigma = \frac{2Q}{4\pi a_1a_2a_3}\left(\frac{x_1^2}{a_1^4} + \frac{x_2^2}{a_2^4} + \frac{x_3^2}{a_3^4}\right)^{-1/2} \tag{21}$$

where x_1, x_2, and x_3 are the coordinates of the ellipsoidal surface. The potential created by this charge is given by

$$U(\xi)=(Q/(4\pi\epsilon)) \cdot \int_0^{\xi}\left[(a_1^2+\eta)(a_2^2+\eta)(a_3^2+\eta)\right]^{-1/2} d\eta \quad (22)$$

where ξ is the ellipsoidal coordinate, which is related to the coordinates x_1, x_2, and x_3 as follows:

$$\left(\frac{x_1^2}{a_1^2+\xi}+\frac{x_2^2}{a_2^2+\xi}+\frac{x_3^2}{a_3^2+\xi}\right)=1 \quad (23)$$

and our reference point corresponds to $\xi=0$, i.e., $U(\xi=0)=0$. At large distances $r \gg a_1, a_2, a_3$, the potential becomes spherically symmetrical and approaches $U_2=U(\xi \to \infty)$. For $a_1 \geqslant a_2 \geqslant a_3$, we can rewrite the equation for U_2 in the following form [5, p. 219]:

$$U_2=(Q/(2\pi\epsilon))(a_1^2-a_2^2)^{-1/2} \cdot F\left\{\cos^{-1}(a_3/a_1),\left[(a_1^2-a_2^2)/(a_1^2-a_3^2)\right]^{1/2}\right\} \quad (24)$$

where F is the elliptic integral of the first kind [5, p. 904]. From (20), we find the voltage drop between the metal and the boundary of the depletion layer:

$$U_1=(-2Q/R_1-2\pi qN_dR_1^2/3)/(4\pi\epsilon)+U_2. \quad (25)$$

The last term, U_2, in (25) is much larger than the other terms (in approximately R_1/a_1 times). Hence, the capacitance is given by

$$C=(dQ/dU_1)\approx(dQ/dU_2)=2\pi\epsilon(a_1^2-a_2^2)^{1/2}/ F\left\{\cos^{-1}(a_3/a_1),\left[(a_1^2-a_2^2)/(a_1^2-a_3^2)\right]^{1/2}\right\}. \quad (26)$$

The error of this expression is of the order of a_1/R_1. In the limiting case of $a_1=a_2=a$, (26) reduces to

$$C=\frac{2\pi\epsilon(a^2-a_3^2)^{1/2}}{\cos^{-1}(a_3/a)} \quad (27)$$

(Here we used the relation $F(x,0)=x$.) In the limiting case of a hemisphere ($a=a_3$), (27) reduces to

$$C=\lim_{a\to a_3}\frac{2\pi\epsilon(a^2-a_3^2)^{1/2}}{\cos^{-1}(a_3/a)}=2\pi\epsilon a \quad (28)$$

which coincides with (9). In the opposite limiting case of a round metal dot on the semiconductor surface ($a_3=0$), (27) yields

$$C=4\epsilon a. \quad (29)$$

As is seen from a comparison of (28) and (29), the capacitances of a hemisphere and a metal dot differ by a shape-dependent numerical factor.

For an elliptic metal dot with an arbitrary relation between the axes, we obtain from (24)

$$C=\frac{2\pi\epsilon a_1}{K\left(\left[1-(a_2/a_1)^2\right]^{1/2}\right)} \quad (30)$$

where K is a complete elliptic integral [5, p. 905]. In the limiting case of an elongated elliptic dot ($a_1 \gg a_2$), (30) reduces to

$$C=\frac{2\pi\epsilon a_1}{\ln(4a_1/a_2)}. \quad (31)$$

Equations (27)–(31) describe the situation when $a_1 \geqslant a_2 \geqslant a_3$. In the opposite case, which corresponds to an elongated ellipsoid embedded into the semiconductor ($a_1 \leqslant a_2 \leqslant a_3$), we obtain

$$C=dQ/dU_1\approx dQ/dU_2=2\pi\epsilon(a_3^2-a_2^2)^{1/2}/ F\left\{\cos^{-1}(a_1/a_3),\left[(a_3^2-a_2^2)/(a_3^2-a_1^2)\right]^{1/2}\right\}. \quad (32)$$

In the limiting case when $a_3 \gg a_2, a_1$, we find

$$C=2\pi\epsilon a_3/(\ln[4a_3/(a_1+a_2)]). \quad (33)$$

When the ellipsoidal cross section is a circle ($a_1=a_2$), we obtain from (32)

$$C=2\pi\epsilon(a_3^2-a^2)^{1/2}\Big/\left\{\ln\left[(a_3^2-a^2)^{1/2}+a_3\right]/a\right\}. \quad (34)$$

The results for different geometries are summarized in Table I.

The results obtained above for a round metal dot apply to the limiting case $R_1 \gg a$. In the opposite case, $R_1 \ll a$, a dot capacitance is given by (11), where, according to the elementary theory of junctions,

$$R_1=(2\epsilon U_1/qN_d)^{1/2}. \quad (35)$$

Below we propose an interpolation formula which may be used at any relation between a and R_1. First of all, we introduce an interpolation for the total charge which is valid in both limiting cases:

$$Q=qN_d\pi(a^2R_1+\beta aR_1^2+2R_1^3/3). \quad (36)$$

The first term in (36) corresponds to the depletion charge under the dot, the second term is the correction arising from the edge charge discussed in Section III, and the third one describes the charge for the case of a very small dot. The interpolation formula for the potential is given by

$$U_1=qN_dR_1^2(1+\pi x^2/[3(x+b_1)])/(2\epsilon). \quad (37)$$

Equations (36) and (37) are chosen in such a way that the resulting expression for the capacitance reproduces the limiting cases considered in Section III and in this section for a small dot. The constant b_1 is an adjustable parameter. This parameter can be determined from a comparison

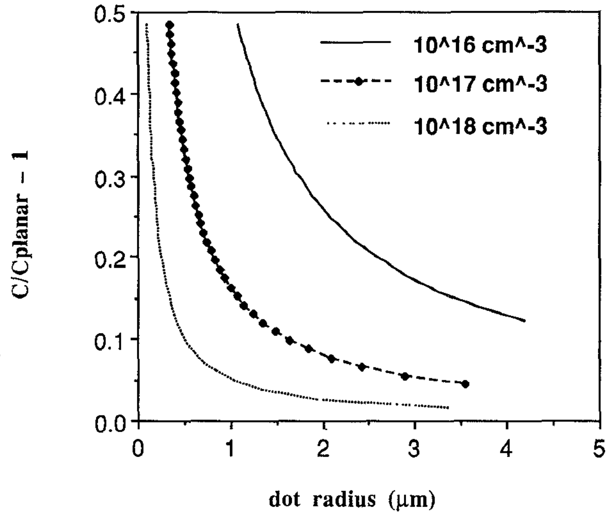


Fig. 2. $C/C_{\text{planar}} - 1$ versus metal dot radius for a GaAs Schottky diode (round dot shape) for different doping densities. The built-in voltage is equal to 0.8 eV, $\epsilon = 1.14\times10^{-10}$ F/m, and $b_1 = 1$.

with numerical calculations. Using (36) and (37), we obtain

$$C = \pi\epsilon a^2(1+2\beta x+2x^2)/\left\{R_1\left[1+\pi x^2(3x+4b_1)/\left(6(x+b_1)^2\right)\right]\right\}. \quad (38)$$

Fig. 2 shows $C/C_{\text{planar}} - 1$ versus a for a GaAs Schottky diode. As can be seen from the figure, the fringing effects are quite important in submicron diodes even for very high doping densities.

VI. Capacitance of a Narrow Metal Stripe on Semiconductor Surface

An approach similar to that used in Section V can be used in order to calculate the capacitance of a narrow metal stripe. We are able to find an analytical solution of this problem only for the limited case of a large depletion layer ($R_1 \gg a$, where a is the half-width of the metal stripe). In this case, the boundary of the depletion layer is still very close to a semicylinder and the total charge in the depletion layer is approximately given by

$$Q = \pi q N_d R_1^2 W/2. \quad (39)$$

Hence, far from the metal stripe ($r \gg a$), the electric potential is approximately the same as for a semicylinder:

$$U = N_d\left(2R_1^2 \ln(r/a) - r^2\right)/(4\epsilon) + U_2. \quad (40)$$

The first and second terms in the parenthesis of (40) describe the contributions from the charge, Q, on the metal surface and from the charge in the depletion layer within a semicylinder of radius r, respectively, and U_2 is a constant to be found. At distances $r \ll R_1$, the depletion charge affects the surface distribution of the electric field only slightly. Therefore, very close to the surface, the solution coincides with that for a metal stripe on a dielectric medium without depletion charges. In the intermediate region, $a \ll r \ll R_1$, the second term in the parentheses on the right-hand side of (40) is much smaller than the first term because the depletion charge may still be neglected. This means that we have to find a solution which coincides with that for a metal stripe on a dielectric medium for $r \ll R_1$ and asymptotically approaches the $\ln(r)$ dependence for large values of r. Following the same approach that was used to derive (22), we obtain the following equation for the potential of a stripe for $r \ll R_1$:

$$U = \left(N_d q R_1^2/(4\epsilon)\right)\int_0^{\xi}\left[\eta(a^2+\eta)\right]^{-1/2} d\eta. \quad (41)$$

Here ξ is the elliptical coordinate, which is related to the coordinates x_2, x_3 as follows:

$$x_3^2/\xi + x_2^2/(\xi + a^2) = 1. \quad (42)$$

The coordinate x_1 is along the stripe and the direction of x_3 is perpendicular to the semiconductor surface. From (41), we find

$$U = \left(N_d q R_1^2/(2\epsilon)\right)\ln\left(\left(\sqrt{\xi}+\sqrt{\xi+a^2}\right)/a\right). \quad (43)$$

At large distances $r \gg a$, the potential becomes cylindrically symmetrical and (43) should reduce to $U = 2N_d R_1^2 \ln(r/a)/(4\epsilon) + U_2$ (compare with (40)). Using this requirement, we can find U_2:

$$U_2 = \left(N_d q R_1^2/(2\epsilon)\right)\ln 2. \quad (44)$$

From (44), we obtain the voltage drop across the depletion layer:

$$U_1 = N_d q R_1^2 \ln\left(\sqrt{y_1}\right)/(2\epsilon) \quad (45)$$

where $y_1 = 4x^2/\exp(1)$. Hence, the capacitance (with a/R_1 accuracy) is given by

$$C = (dQ/dU_1) \approx \epsilon\pi W/\ln\left[\exp(1)\sqrt{y_1}\right]. \quad (46)$$

As may be expected, this equation is fairly similar to (16) for the capacitance of a metal semicylinder embedded in a semiconductor. In both cases, $C = \epsilon\pi W/\ln(\text{Const}\, R_1/a)$ and only the constant is different. Hence, we expect that the same expression is applicable for a metal stripe of an arbitrary cross section embedded in a semiconductor.

The results obtained above for a metal stripe apply to the limiting case $R_1 \gg a$. In the opposite case, $R_1 \ll a$, the stripe capacitance is given by (11). Below we propose an interpolation formula which may be used for any relation between a and R_1. First, we introduce an interpolation for the total charge which is valid in both limiting cases:

$$Q = 2qN_d W a R_1\{1 + x[(b_2 + b_c/4) + b_2\pi x/2]/(1 + b_2 x)\} \quad (47)$$

where $b_c = 2(W+2a)\beta/W$ and the constant b_2 is an adjustable parameter. The first term in (47) corresponds to the depletion charge under the dot, the second term is the correction caused by the edge charge, and the third term describes the charge for the case of a very narrow stripe.

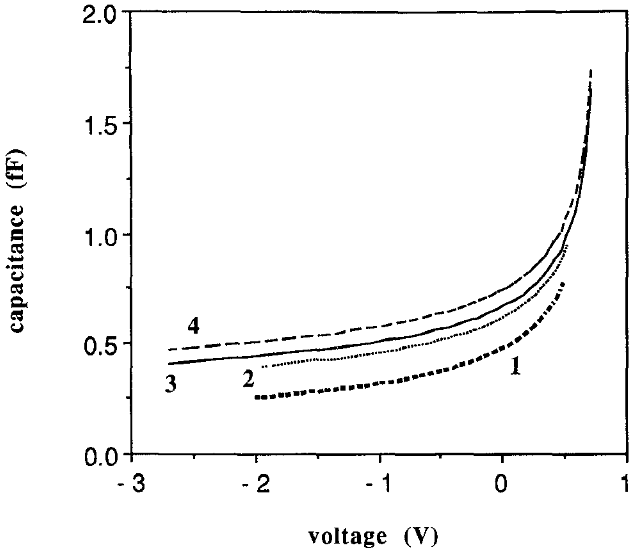


Fig. 3. C versus a bias for GaAs Schottky diodes (stripes and round dot). Curve *1* is calculated by using the planar capacitance formula; curve *2* corresponds to a metal dot ($a = 0.25$ μm); curve 3 corresponds to a square ($W/a = 2$); curve *4* corresponds to a rectangle ($W/a = 4$). The built-in voltage is equal to 0.8 eV, $\epsilon = 1.14 \times 10^{-10}$ F/m, $b_2 = 1$, and $N_d = 5 \times 10^{17}$ cm^{-3}.

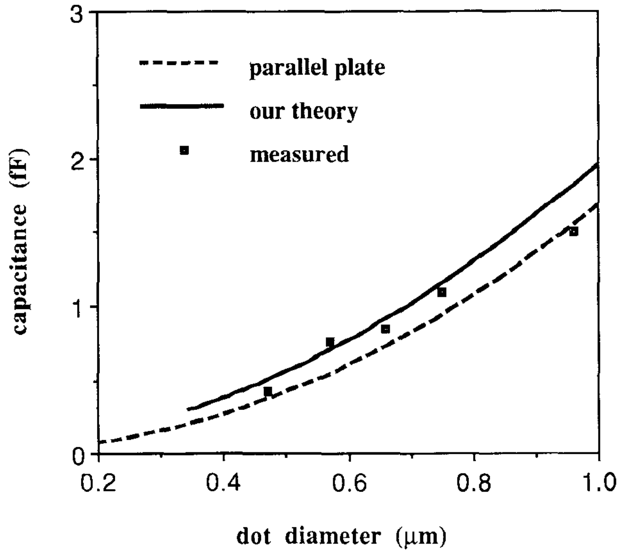


Fig. 4. Measured and calculated dependences of the round-dot GaAs Schottky barrier diode capacitance on the dot diameter. The top curve is calculated by using our theory. The bottom curve is calculated by using the planar capacitance formula. Doping density, 4×10^{17} cm^{-3}; zero applied voltage; built-in voltage, 0.8 eV. The crosses denote measured data.

The interpolation formula for the potential is given by

$$U_1 = qN_dR_1^2[1 + \ln(1 + y_1)/2]/(2\epsilon). \tag{48}$$

Equations (47) and (48) are chosen in such a way that the resulting expression for the capacitance reproduces the limiting cases considered in Section IV and in this section for a narrow metal stripe. The constant b_2 can be determined from a comparison with numerical calculations. Using (47) and (48), we obtain

$$C = 2\epsilon Wa \frac{1 + (3\pi b_2 x + b_c)x/[2(1 + b_2 x)] - b_2(2\pi b_2 x + b_c)\{x/[2(1 + b_2 x)]\}^2}{R_1\{1 + \ln[1 + y_1]/2 + y_1/[2(1 + y_1)]\}}. \tag{49}$$

Fig. 3 shows $C-V$ dependences for GaAs Schottky diodes of the same area but of different geometries. As can be seen from the figure, the round metal dot has the smallest fringing capacitance because it has the smallest perimeter. The fringing capacitance increases for longer and narrower stripes. The capacitance modulation is the largest for the round dot.

VII. Results and Discussion

Our results can be applied to a large variety of Schottky diodes (see Table I). Our theory is also applicable to p–n junctions. Most Schottky diodes have shapes of metal dots or metal stripes. However, other shapes may be relevant, for example, for diffused or ion-implanted p–n junctions. Also, with a trivial change ($C \to 1/R$ and $\epsilon \to \sigma$, where σ is the conductivity and R the resistance), our results can be applied for calculating resistances for different geometries.

We measured $C-V$ characteristics of round-dot GaAs Schottky barrier diodes by registering the capacitance between a metal whisker and the wafer with Schottky diodes and determining the capacitance jump when the whisker touches the Schottky contact. The experimental data are compared with our theory in Fig. 4. As can be seen from the figure, the agreement with the experimental data is quite reasonable.

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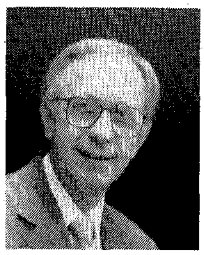

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